Name ______________________________ Period ________

Math Properties

	Property	Example
1.	Commutative Property of Addition $a + b = b + a$	$2 + 3 = 3 + 2$
2.	Commutative Property of Multiplication $a \bullet b = b \bullet a$	$2 \bullet (3) = 3 \bullet (2)$
3.	Associative Property of Addition $a + (b + c) = (a + b) + c$	$2 + (3 + 4) = (2 + 3) + 4$
4.	Associative Property of Multiplication $a \bullet (b \bullet c) = (a \bullet b) \bullet c$	$2 \bullet (3 \bullet 4) = (2 \bullet 3) \bullet 4$
5.	Distributive Property $a \bullet (b + c) = a \bullet b + a \bullet c$	$2 \bullet (3 + 4) = 2 \bullet 3 + 2 \bullet 4$
6.	Additive Identity Property $a + 0 = a$	$3 + 0 = 3$
7.	Multiplicative Identity Property $a \bullet 1 = a$	$3 \bullet 1 = 3$
8.	Additive Inverse Property $a + (-a) = 0$	$3 + (-3) = 0$
9.	Multiplicative Inverse Property $a \bullet \left(\frac{1}{a}\right) = 1$ **Note:** a cannot = 0	$3 \bullet \left(\frac{1}{3}\right) = 1$
10.	Zero Property $a \bullet 0 = 0$	$5 \bullet 0 = 0$

Definitions for Properties of Mathematics

Associative Property of Addition

When three or more numbers are added, the sum is the same regardless of the grouping of the addends. For example (a + b) + c = a + (b + c)

Associative Property of Multiplication

When three or more numbers are multiplied, the product is the same regardless of the ~~order~~ grouping of the multiplicands. For example (a x b) x c = a x (b x c)

Commutative Property of Addition

When two numbers are added, the sum is the same regardless of the order of the addends. For example a + b = b + a

Commutative Property of Multiplication

When two numbers are multiplied together, the product is the same regardless of the order of the multiplicands. For example a x b = b x a

Distributive Property

The sum of two numbers times a third number is equal to the sum of each addend times the third number. For example a x (b + c) = a x b + a x c

Identity Property of Addition

The sum of any number and zero is the original number. For example a + 0 = a.

Identity Property of Multiplication

The product of any number and one is that number. For example a x 1 = a.

Additive Inverse of a Number

The additive inverse of a number, a is -a so that a + -a = 0.

Multiplicative Inverse of a Number

The multiplicative inverse of a number, a is $\frac{1}{a}$ so that a x $\frac{1}{a}$ = 1.

Addition Property of Zero

Adding 0 to any number leaves it unchanged. For example a + 0 = a.

Multiplication Property of Zero

Multiplying any number by 0 yields 0. For example a x 0 = 0.

Properties of Basic Mathematical Operations

Some mathematical operations have properties that can make them easier to work with and can actually save you time.

Some properties (axioms) of addition

You should know the definition of each of the following properties of addition and how each can be used.

- **Closure** is when all answers fall into the original set. If you add two even numbers, the answer is still an even number (2 + 4 = 6); therefore, the set of even numbers *is closed* under addition (has closure). If you add two odd numbers, the answer is not an odd number (3 + 5 = 8); therefore, the set of odd numbers is *not closed*under addition (no closure).
- **Commutative** means that the *order* does not make any difference in the result.

  $$2+3=3+2$$

  $$a+b=b+a$$

  Note: Commutative does not hold for subtraction.

  $$3-1\neq 1-3$$

  $$2\neq -2$$

  $$a-b\neq b-a$$

- **Associative** means that the *grouping* does not make any difference in the result.

  $$(2+3)+4=2+(3+4)$$

  $$(a+b)+c=a+(b+c)$$

  The grouping has changed (parentheses moved), but the sides are still equal.

  Note: Associative does *not* hold for subtraction.

$$4-(3-1)\neq(4-3)-1$$
$$4-2\neq1-1$$
$$2\neq0$$
$$a-(b-c)\neq(a-b)-c$$

- The **identity element** for addition is 0. Any number added to 0 gives the original number.

$$3+0=0+3=3$$
$$a+0=0+a=a$$

- The **additive inverse** is the opposite (negative) of the number. Any number plus its additive inverse equals 0 (the identity).

$3+(-3)=0$; therefore, 3 and -3 are additive inverses.

$-2+2=0$; therefore, -2 and 2 are additive inverses.

$a+(-a)=0$; therefore, a and $-a$ are additive inverses.

Some properties (axioms) of multiplication

You should know the definition of each of the following properties of multiplication and how each can be used.

- **Closure** is when all answers fall into the original set. If you multiply two even numbers, the answer is still an even number (2 × 4 = 8); therefore, the set of even numbers *is closed* under multiplication (has closure). If you multiply two odd numbers, the answer is an odd number (3 × 5 = 15); therefore, the set of odd numbers *is closed* under multiplication (has closure).
- **Commutative** means the *order* does not make any difference.

$$2\times3=3\times2$$
$$a\times b=b\times a$$

Note: Commutative does *not* hold for division.

$$2 \div 4 \neq 4 \div 2$$

$$\frac{2}{4} \neq \frac{4}{2}$$

$$\frac{1}{2} \neq 2$$

$$a \div b \neq b \div a$$

- **Associative** means that the *grouping* does not make any difference.

$$(2 \times 3) \times 4 = 2 \times (3 \times 4)$$

$$(a \times b) \times c = a \times (b \times c)$$

The grouping has changed (parentheses moved) but the sides are still equal.

Note: Associative does *not* hold for division.

$$(8 \div 4) \div 2 \neq 8 \div (4 \div 2)$$

$$2 \div 2 \neq 8 \div 2$$

$$1 \neq 4$$

$$(a \div b) \div c \neq a \div (b \div c)$$

- The **identity element** for multiplication is 1. Any number multiplied by 1 gives the original number.

$$3 \times 1 = 1 \times 3 = 3$$

$$a \times 1 = 1 \times a = a$$

- The **multiplicative inverse** is the **reciprocal** of the number. Any nonzero number multiplied by its reciprocal equals 1.

$2 \times \frac{1}{2} = 1$; therefore, 2 and $\frac{1}{2}$ are multiplicative inverses.

$a \times \frac{1}{a} = 1$; therefore, *a* and $\frac{1}{a}$ are multiplicative inverses (provided a 0).

A property of two operations

The distributive property is the process of passing the number value outside of the parentheses, using multiplication, to the numbers being added or subtracted inside the parentheses. In order to apply the distributive property, it must be multiplication outside the parentheses and either addition or subtraction inside the parentheses.

$$2(3+4)=2(3)+2(4)$$
$$2(7)=6+8$$
$$14=14$$
$$a(b+c)=a(b)+a(c)$$

$$5(12-3)=5(12)-5(3)$$
$$5(9)=60-15$$
$$45=45$$
$$a(b-c)=a(b)-a(c)$$

Note: You cannot use the distributive property with only one operation.

$$3(4\times5\times6)\neq3(4)\times3(5)\times3(6)$$
$$3(120)\neq12\times15\times18$$
$$360\neq3240$$
$$a(bcd)\neq a(b)\times a(c)\times a(d) \text{ or}$$
$$a(bcd)\neq(ab)(ac)(ad)$$