Unit Rate as the Constant of Proportionality

If a proportion is described by the set of ordered pairs that satisfies the equation $y = kx$, where k is called the constant of proportionality.

Constant of Proportionality = unit rate = k

Example

Example : You Need WHAT???

Brandon came home from school and informed his mother that he had volunteered to make cookies for his entire grade level. He needed 3 cookies for each of the 96 students in 7th grade. Unfortunately, he needed the cookies for an event at school on the very next day! Brandon and his mother determined that they can fit 36 cookies on two cookie sheets. *Encourage students to make a chart to organize the data from the problem.*

a. Is the number of cookies proportional to the number of sheets used in baking? Create a table that shows data for the number of sheets needed for the total number of cookies needed.

Table:

# of cookie sheets	*# of cookies baked*	
2	36	$\frac{36}{2} = 18$
4	72	$\frac{72}{4} = 18$
10	180	$\frac{180}{10} = 18$
16	288	$\frac{288}{16} = 18$

The unit rate is 18

The constant of proportionality is 18

Meaning of Constant of Proportionality in this problem: There are 18 cookies per 1 sheet.

b. It took 2 hours to bake 8 sheets of cookies. If Brandon and his mother begin baking at 4:00 pm, when will they finish baking the cookies?

$$\frac{\frac{96 \text{ students } (3 \text{ cookies each})}{288 \text{ cookies}}}{18 \text{ cookies per sheet}} = 16 \text{ sheets of cookies}$$

It takes 2 hours to bake 8 sheets, it will take 4 hours to bake 16 sheets of cookies. They will finish baking at 8:00pm.

Lesson Summary

If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where k is a positive constant, then k is called the *constant of proportionality*.

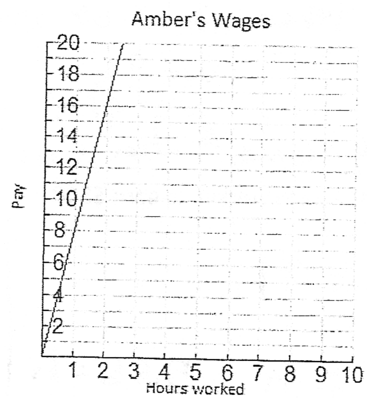
Representing Proportional Relationships with Equations

How can we use what we know about the constant of proportionality to represent proportional relationships by equations.

Write an equation that will model the real world situation.

John and Amber work at an ice cream shop. The hours worked and wages earned are given for each person.

John's wages	
Time (h)	Wages ($)
2	18
3	27
4	36



a. Determine whether John's wages are proportional to time. If they are, determine the unit rate. If not, explain why not.

Yes, the unit rate is 9. The ratios in the table are equivalent.

b. Determine whether Amber's wages are proportional to time. If they are, determine the unit rate. If not, explain why not.

Yes, the unit rate is 8. The collection of ratios is equivalent.

c. Write an equation to model the relationship between each person's wages. Identify constant of proportionality for each. Explain what it means in the context of the situation.

John $W = 9h$ constant of prop. $= 9$ John earns \$9 for every hour he works.

Amber $W = 8h$ Constant of prop. $= 8$ Amber earns \$8 for every hour she works

d. How much would each worker make after working 10 hours? Who will earn more money?

After 10 hours John will earn \$90 because 10 hours is the value of the independent variable which should be multiplied by K (the constant of proportionality)

$W = 9h$

$W = 9(10)$ $W = 90$

after 10 hours Amber will earn \$80

$A = 8h$

$A = 8(10) = 80$

$\boxed{\text{John earns more}}$

e. How long will it take each worker to earn \$50?

$50 = 9h$

$\frac{50}{9} = h$ $5.56 = h$

It will take John 5.56 hours to earn \$50

$50 = 8h$

$\frac{50}{8} = h$

$6.25 = h$

It will take Amber 6.25 hours to earn \$50

Lesson Summary:

The points $(0, 0)$ and $(1, r)$, where r is the unit rate, will always fall on the line representing two quantities that are proportional to each other.

The unit rate r in the point $(1, r)$ represents the amount of vertical increase for every horizontal increase of 1 unit on the graph.

The point $(0, 0)$ indicates that when there is zero amount of one quantity, there will also be zero amount of the second quantity.

These two points may not always be given as part of the set of data for a given real-world or mathematical situation, but they will always fall on the line that passes through the given data points.

1. A person who weighs 100 pounds on Earth weighs 16.6 lb. on the moon.

a. Which variable is the independent variable? Explain why.

Weight on the earth is the independent variable because most people do not fly to the moon to weigh themselves first. The weight on the moon depends on a person's weight on the earth.

b. What is an equation that relates weight on Earth to weight on the moon?

$m = \frac{16.6}{100} E$

c. How much would a 185 pound astronaut weigh on the moon?

30.71 lb.

d. How much would a man that weighed 50 pounds on the moon weigh back on Earth?

301 lb. $y = mx$

2. Use this table to answer the following questions.

Gallons	Miles driven
0	0
2	62
4	124
10	310

a. Which variable is the dependent variable and why?

The # of miles driven is the dependent variable because the # of miles you can drive depends on the # of gallons of gas you have in your tank.

b. Is miles driven proportionally related to gallons? If so, what is the equation that relates miles driven to gallons?

Yes, miles driven is proportionally related to gallons because every measure of gallons can be multiplied by 31 to get every corresponding measure of miles driven

$m = 31G$

c. In any ratio relating gallons and miles driven, will one of the values always be larger, if so, which one?

Yes, miles

d. If the number of gallons is known, can you find the miles driven? Explain how this value would be calculated?

Yes, multiply the constant of proportionality (31 mpg) by the # of gallons

e. If the number of miles driven is known, can you find the number of gallons consumed?

Explain how this value would be calculated?

Yes, divide the # of miles driven by constant of prop. (31 mpg)

f. How many miles could be driven with 18 gallons of gas?

558

g. How many gallons are used when the car has been driven 18 miles?

18/31 of a gallon

h. How many miles have been driven when ½ of a gallon is used?

$\frac{31}{2}$ = 15.5 miles

i. How many gallons have been used when the car has been driven ½ mile?

$\frac{1}{62}$ of a gallon

3. Suppose that the cost of renting a snowmobile is $37.50 for 5 hours.

a. If the c = cost and h = hours, which variable is the dependent variable? Explain why.

C is the dependent variable because the cost of using the snowmobile depends on the # of hours you use it.

C = 7.5h

b. What would be the cost of renting 2 snow mobiles for 5 hours each?

$75

4. In mom's car, the number of miles driven is proportional to the number of gallons of gas used.

Gallons	Miles driven
0	0
4	112
6	168
8	224
10	280

a. Write the equation that will relate the number of miles driven to the gallons of gas.

M = 28G

b. What is the constant of proportionality?

28

c. How many miles could you go if you filled your 22-gallon tank?

616 miles

d. If your family takes a trip of 600 miles, how many gallons of gas would be needed to make the trip?

$21\frac{3}{7}$ gallons

e. If you drive 224 miles during one week of commuting to school and work, how many gallons of gas would you use?

8 gallons