# 6.3 The Probability of Two Events

In this section we review the use of *listings*, *tables* and *tree diagrams* to calculate the probabilities of two events.

## Example 1

An unbiased coin is tossed twice.

(a) List *all* the possible outcomes.

(b) What is the probability of obtaining *two heads*?

(c) What is the probability of obtaining a *head* and a *tail* in any order?

### Solution

(a) The possible outcomes are:

H H

H T

T H

T T

So there are 4 possible outcomes that are all equally likely to occur as the coin is not biased.

(b) There is only one way of obtaining 2 heads, so:

$$p(\text{2 heads}) = \frac{1}{4}$$

(c) There are two ways of obtaining a head and a tail, H T and T H, so:

$$p(\text{a head and a tail}) = \frac{2}{4} = \frac{1}{2}$$

## Example 2

A red dice and a blue dice, both unbiased, are rolled at the same time. The scores on the two dice are then added together.

(a) Use a table to show all the possible outcomes.

(b) What is the probability of obtaining:

(i) a score of 5,

(ii) a score which is *greater than* 3,

(iii) a score which is an *even number*?

## Solution

(a) The following table shows all of the 36 possible outcomes:

| | | *Red Dice* | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| *Blue Dice* | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

(b) (i) There are 4 ways of scoring 5, so:

$$p(5) = \frac{4}{36} = \frac{1}{9}$$

(ii) There are 33 ways of obtaining a score greater than 3, so:

$$p(\text{greater than } 3) = \frac{33}{36} = \frac{11}{12}$$

(iii) There are 18 ways of obtaining a score which is an even number, so:

$$p(\text{even score}) = \frac{18}{36} = \frac{1}{2}$$

## Example 3

A card is taken at random from a pack of 52 playing cards, and then replaced. A second card is then drawn at random from the pack.

Use a tree diagram to determine the probability that:

(a) *both* cards are Diamonds,

(b) *at least one* card is a Diamond,

(c) *exactly one* card is a Diamond,

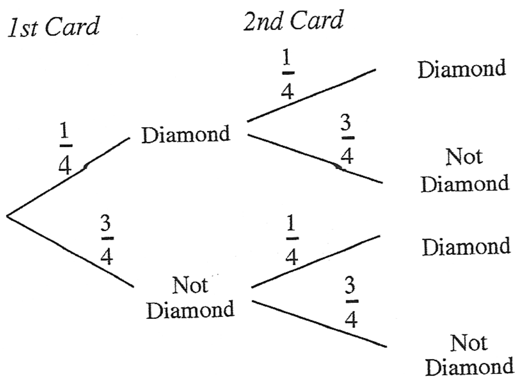
(d) *neither* card is a Diamond.

## Solution

We first note that, for a single card drawn from the pack,

$$p(\text{Diamond}) = \frac{13}{52} = \frac{1}{4} \text{ and } p(\text{not Diamond}) = \frac{39}{52} = \frac{3}{4}.$$

We put these probabilities on the branches of the tree diagram below:




*PROBABILITIES*

$$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

$$\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$$

$$\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$$

$$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

$$\text{Total} = \frac{16}{16} = 1$$

Note also that the probability for each combination, for example, two Diamonds, is determined by *multiplying* the probabilities along the branches.

(a) $p(\text{both Diamonds}) = \frac{1}{16}$

(b) $p(\text{at least one Diamond}) = \frac{1}{16} + \frac{3}{16} + \frac{3}{16}$

$$= \frac{7}{16}$$

(c) $p(\text{exactly one Diamond}) = \frac{3}{16} + \frac{3}{16}$

$$= \frac{6}{16}$$

$$= \frac{3}{8}$$

(d) $p(\text{neither card a Diamond}) = \frac{9}{16}$