

6.2 The Probability of a Single Event

In this section we consider the probabilities of equally likely events. When you roll a fair dice, each of the numbers 1 to 6 is equally likely to be on the uppermost face of the dice.

For equally likely events:

$$p(\text{a particular outcome}) = \frac{\text{number of ways of obtaining outcome}}{\text{total number of outcomes}}$$

Example 1

A card is taken at random from a full pack of 52 playing cards. What is the probability that it is:

- (a) a red card,
- (b) a 'Queen',
- (c) a red 'Ace',
- (d) the 'Seven of Hearts',
- (e) an even number?

Solution

As each card is equally likely to be drawn from the pack there are 52 equally likely outcomes.

- (a) There are 26 red cards in the pack, so:

$$\begin{aligned} p(\text{red}) &= \frac{26}{52} \\ &= \frac{1}{2} \end{aligned}$$

- (b) There are 4 Queens in the pack, so:

$$\begin{aligned} p(\text{Queen}) &= \frac{4}{52} \\ &= \frac{1}{13} \end{aligned}$$

- (c) There are 2 red Aces in the pack, so:

$$\begin{aligned} p(\text{red Ace}) &= \frac{2}{52} \\ &= \frac{1}{26} \end{aligned}$$

(d) There is only one 7 of Hearts in the pack, so:

$$p(7 \text{ of Hearts}) = \frac{1}{52}$$

(e) There are 20 cards that have even numbers in the pack, so:

$$\begin{aligned} p(\text{even number}) &= \frac{20}{52} \\ &= \frac{5}{13} \end{aligned}$$

Example 2

A packet of sweets contains 18 *red* sweets, 12 *green* sweets and 10 *yellow* sweets. A sweet is taken at random from the packet. What is the probability that the sweet is:

- (a) *red*,
- (b) *not green*,
- (c) *green* or *yellow*?

Solution

The total number of sweets in the packet is 40, so there are 40 equally likely outcomes when one is taken at random.

(a) There are 18 *red* sweets in the packet, so:

$$\begin{aligned} p(\text{red}) &= \frac{18}{40} \\ &= \frac{9}{20} \end{aligned}$$

(b) There are 28 sweets that are *not green* in the packet, so:

$$\begin{aligned} p(\text{not green}) &= \frac{28}{40} \\ &= \frac{7}{10} \end{aligned}$$

(c) There are 22 sweets that are *green* or *yellow* in the packet, so:

$$\begin{aligned} p(\text{green or yellow}) &= \frac{22}{40} \\ &= \frac{11}{20} \end{aligned}$$

Example 3

You roll a fair dice 120 times. How many times would you expect to obtain:

- (a) a 6, (b) an *even* score, (c) a score of *less than 5*?

Solution

$$(a) \quad p(6) = \frac{1}{6}$$

$$\begin{aligned} \text{Expected number of 6s} &= \frac{1}{6} \times 120 \\ &= 20 \end{aligned}$$

$$\begin{aligned} (b) \quad p(\text{even score}) &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{Expected number of even scores} &= \frac{1}{2} \times 120 \\ &= 60 \end{aligned}$$

$$\begin{aligned} (c) \quad p(\text{score less than 5}) &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{Expected number of scores less than 5} &= \frac{2}{3} \times 120 \\ &= 80 \end{aligned}$$