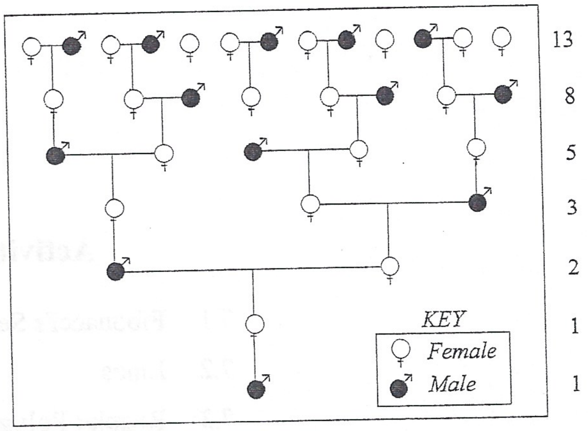
ACTIVITY 7.1 *Fibonacci's Sequence*

The Italian mathematician, *Leonardo de Pisano* (nicknamed *Fibonacci*) lived from about AD1170 to 1250. He devoted much of his time and effort to the study of the so-called *Fibonacci numbers*:

1, 1, 2, 3, 5, 8, 13, 21, 34, . . .

This sequence appears frequently in the natural world; for example, in the pattern of reproduction of bees (shown opposite), the arrangements of leaves on stems, petals on flowers and spirals on cones.



Pattern of bee reproduction

1. Write down the next seven numbers in the sequence.

2. Consider the first *four* numbers in the sequence

   1, 1, 2, 3

   Multiply the two outside numbers together and then multiply the two inside numbers together. What is the difference?

   Repeat this procedure for any four consecutive numbers in Fibonacci's sequence.

   What do you notice?

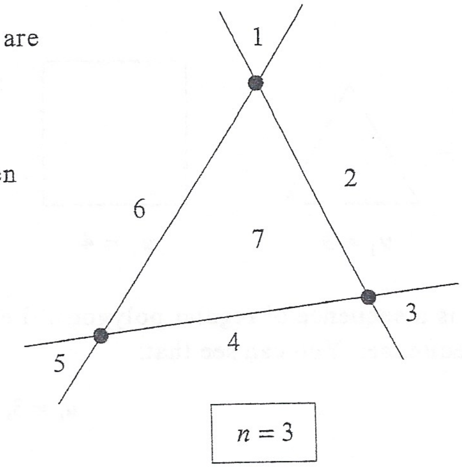
Extension
Use the same rule for generating the sequence, but start with

(a) 2, 5 (b) 1, 3.

In each case, find the next seven terms.

ACTIVITY 7.2 *Lines*

If three lines are arranged as in the diagram, there are seven regions formed, with three crossover points.



This investigation looks at the relationship between

- the number of lines, n

and the maximum number of

- crossover points
- regions.

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1. Draw similar diagrams to find the maximum number of crossover points and regions for:

   (a) 2 lines (b) 4 lines (c) 5 lines.

2. Predict the result for:

   (a) 6 lines (b) 7 lines.

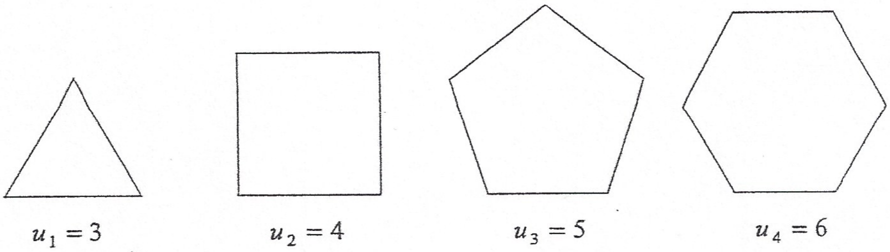
Extension

(a) Generalise your results and write down formulae for the maximum number of crossovers and regions.

(b) Use the formulae to predict the maximum number of crossover points and regions for:

(i) 20 lines (ii) 100 lines.

ACTIVITY 7.3 *Regular Polygons*



Here is a sequence of regular polygons. Let u_n be the number of sides of the nth shape in the sequence. You can see that:

$$u_1 = 3, \quad u_2 = 4, \ \ldots$$

1. What is the value of u_5 and u_6?
2. What is the general formula for u_n? Check your answer for u_7.
3. How many diagonals can be drawn from a single vertex in each of the shapes above?
4. How many diagonals can be drawn from a vertex of the nth shape in the sequence?
5. How many diagonals in total can be drawn in each of the shapes above?

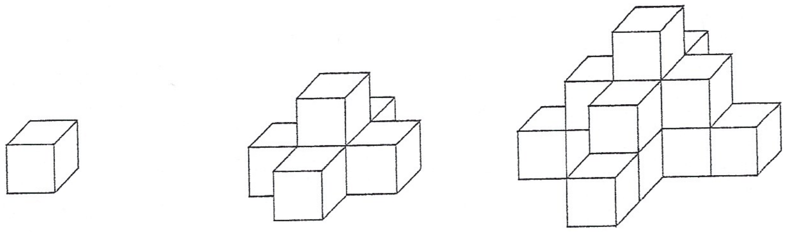
Extension

How many diagonals in total can be drawn in the nth shape?

ACTIVITY 7.4

Towers

We will investigate how many cubes are needed to build towers like these, where the 'towers' are amde up of a series of steps up to the pinnacle.



1 2 3

1. How many cubes are needed for each of the towers above?

2. For tower '3', how many cubes are needed for each layer?

3. For tower '4', how many cubes are needed for the bottom layer? How many are needed for the whole tower?

You can present the information and predict how many cubes are needed by taking first and second differences, as shown below. First copy the table; include your answers to questions 2 and 3 together with any other information this leads to.

Tower No.	*1*		*2*		*3*		*4*		*5*		*6*
No. of cubes needed	1		6		?		?		?		?
Ist difference		5		?		?		x		y	
2nd difference			4		4		4		4		

4. If the 2nd difference remains constant, what is the value of

   (a) x (b) y ?

5. Hence deduce the number of cubes neede for tower '5' and tower '6'.

Extension
How many cubes are needed for tower '10' ?