Lesson
24 Draw Shapes that Meet Given Conditions

Essential Question:
How can you construct geometric shapes with given conditions?
7.G.2

Guided Instruction

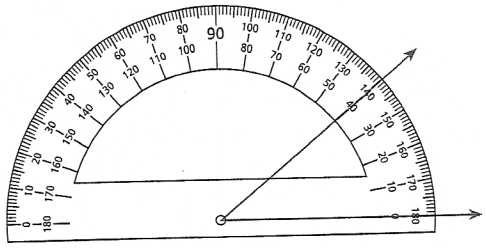
In this lesson, you will learn how to draw shapes that meet given conditions. You will learn how to tell if three side lengths or three angle measures can be used to draw *exactly one triangle*, *more than one triangle*, or *no triangle*.

Understand: Drawing a geometric shape to meet given conditions

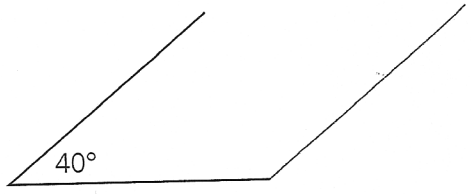
> Lamar is asked to draw a quadrilateral that has exactly one pair of parallel sides and one angle that measures 40°. What might this quadrilateral look like?

Lamar follows these steps to draw his shape.

Lamar uses a protractor to draw the 40° angle. The sides of the angle become two sides of the quadrilateral.

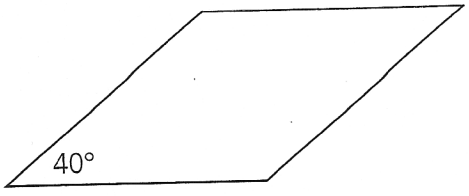


Lamar uses a ruler to draw a third side parallel to one of the sides of the angle.

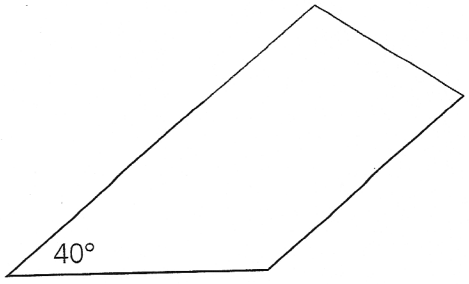


Lamar draws the fourth side of the quadrilateral.

Lamar first draws this line segment as the fourth side. Then he realizes that his shape now has two sets of parallel sides instead of just one set.



Lamar revises his drawing.



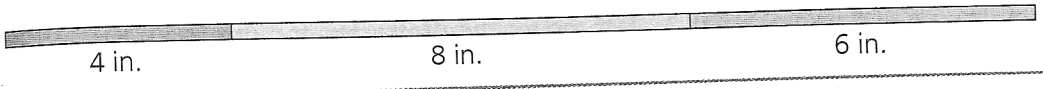
The second figure at the right shows the drawing that Lamar made to meet the given conditions.

- Draw a figure that is different from Lamar's, but still meets the given conditions. Use a ruler and a protractor or technology. What is the best name for the shape?

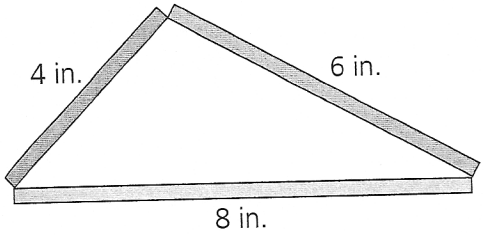
Understand: Drawing a triangle when given three side lengths

Stan has three straws that are 4 inches, 8 inches, and 6 inches long. Can he make a triangle using the straws? Can he make *more than one triangle*?

Stan tries to make a triangle. He lays out the straws side by side.



Stan makes a triangle by moving the red and purple straws so that they meet at a point.



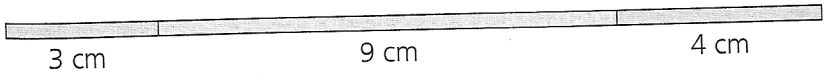
This is the only triangle that Stan can make using the three straws. If Stan makes a triangle that looks different, he can turn it or flip it so that it exactly fits onto this triangle.

We say that the three sides make *exactly one triangle* or that the three sides determine *one unique triangle*.

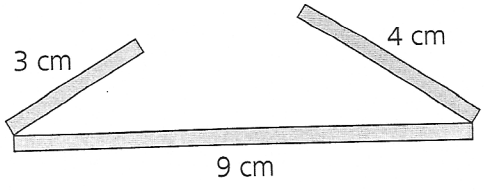
➧ Stan can make *exactly one triangle* using straws with lengths 4 in., 8 in., and 6 in.

Stan has three more straws that are 3 centimeters, 9 centimeters, and 4 centimeters long. Can he make a triangle using those straws?

Stan tries to make a triangle. He lays out the straws side by side.



Stan tries to connect the teal and green straws but they are too short. He cannot make a triangle.



➧ Stan can make *no triangle* using straws with lengths 3 cm, 9 cm, and 4 cm.

These examples help illustrate a property of triangles.

Property of Triangles: Given 3 side lengths, if each side length is less than the sum of the other two side lengths, *exactly one triangle* can be constructed. If any one side length is greater than or equal to the sum of the other two side lengths, *no triangle* can be constructed.

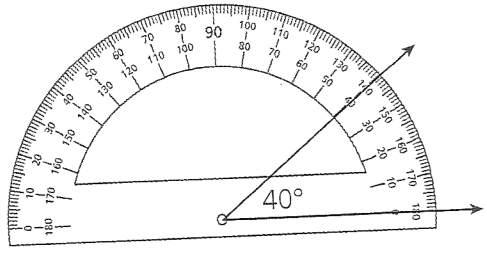
Guided Instruction

Understand: Drawing a triangle when given three angle measures

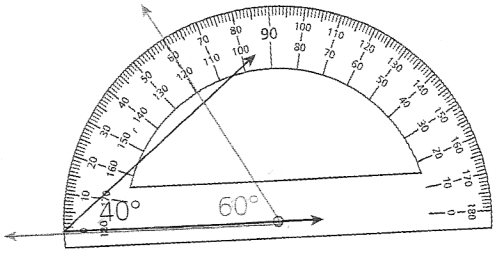
Can Jeff make a triangle with these three angle measures: 40°, 60°, and 80°?
Can he make *more than one triangle*?

Here are the steps Jeff uses to try to make a triangle.

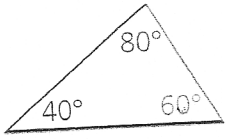
Jeff uses a protractor to draw a 40° angle.



Jeff moves the protractor and draws a 60° angle that shares a ray with the 40° angle.

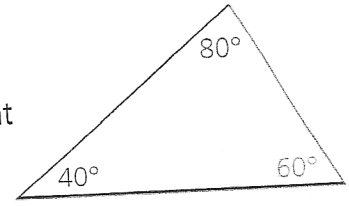


The rays intersect to form a triangle with a 40° angle and a 60° angle. Jeff measures the third angle. It measures 80°.



Jeff has made a triangle with angle measures 40°, 60°, and 80°.

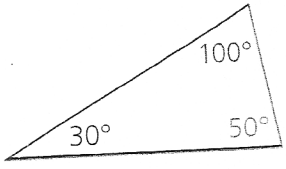
Jeff realizes that he can make a different triangle by keeping the angles the same measure and increasing the lengths of the sides. He sees that there is no limit to the number of triangles he can make. We can say that *more than one triangle* can be made. We can also say that *an infinite number of triangles* can be made.



➡ Jeff can make *more than one triangle*.

Can Jeff make a triangle with these three angle measures: 30°, 50°, and 25°?

Jeff uses the same method as in the first example. But he finds that the third angle has a measure of 100°, not 25°. Jeff cannot make a triangle with angle measures 30°, 50°, and 25°.



➡ Jeff can make *no triangle*.

The above examples help to illustrate this property of triangles.

Property of Triangles: If the sum of three angle measures is exactly 180°, *an infinite number of triangles* can be constructed. If the sum is less than or greater than 180°, *no triangle* can be constructed.

Connect: Constructing triangles when given 3 side lengths or 3 angle measures

When given 3 side lengths or 3 angle measures, how do you predict if you can construct *exactly one triangle*, *more than one triangle*, or *no triangle* for each set of conditions?

Under certain conditions you can make general statements about constructing triangles.

The examples on pages 213 and 214 illustrate these four **Properties of Triangles:**

Given 3 side lengths:

1. If each side length is less than the sum of the other two side lengths, *exactly one triangle* can be constructed.
2. If any one side length is greater than or equal to the sum of the other two side lengths, *no triangle* can be constructed.

Given 3 angle measures:

3. If the sum of the angle measures is exactly 180°, *more than one triangle* can be constructed. In fact, an infinite number of triangles can be constructed.
4. If the sum of the angle measures is greater than or less than 180°, *no triangle* can be constructed.

• Use the properties of triangles to predict whether you can draw *exactly one triangle*, *more than one triangle*, or *no triangle* for each set of conditions. Then try to draw the triangle to check your prediction.

a. side lengths: 3 cm, 4 cm, 5 cm

Predict

_____

Property used: ____

Draw

Was your prediction correct? _____

b. angle measures: 75°, 25°, 30°

Predict

_____

Property used: ____

Draw

Was your prediction correct? _____